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FREE CONVECTION HEAT TRANSFER TO MERCURY IN VERTICAL ANNULI

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Abstract—Data on free convection heat transfer to water and mercury are collected using a test rig in vertical annuli of three radii ratios, the walls of which are maintained at uniform temperatures. A theoretical analysis of the boundary layer equations has been attempted using local similarity transformation and double boundary layer approach. Correlations derived from the present theoretical analysis are compared with the analysis and the experimental data available in literature for non-metallic fluids and also with the present experimental data on water and mercury. Generalised correlations are set up for expressing the ratio of heat transferred by convection to the heat transferred by pure conduction and Nusselt's number, in terms of Grashof, Rayleigh and Prandtl numbers, based on the theoretical analysis and the present data on mercury and water. The present generalised correlations agree with the reported and present data for non-metallic fluids and liquid metals with an average deviation of 9% and maximum deviation of ± 13.7%.

NOMENCLATURE

A, constant;
C_P, specific heat;
C, radii ratio;
D, diameter;
D_0, outer diameter of inner cylinder;
D_1, inner diameter of outer cylinder;
d, radii gap of annuli (constant);
f, function of similarity variable;
g, gravitational constant;
Gr, Grashof number;
h, heat transfer coefficient;
K_f, thermal conductivity of fluid;
L, height of annulus;
Nu, Nusselt number;
N, P, slope and intercept;
Pr, Prandtl number;
Q, total heat;
R, r, radial coordinates;
R_0, outer radius of inner cylinder;
Ra, Rayleigh number;
s, S, slope and intercept;
t, T, temperature;
u, V, non-dimensional velocities;
$\bar{u}, \bar{v}$, dimensional velocities;
W, mass rate of flow;
X, coordinate measuring distance from leading edge;
Y, y, coordinate measuring distance perpendicular to surface.

Greek symbols

α, thermal diffusivity;
β, coefficient of volumetric expansion;
ρ, density;
ν, kinematic viscosity;
η, similarity variable;
θ, dimensionless temperature;
ψ, stream function.

Subscripts

W, condition at wall;
1, 2, condition at hot and cold surface;
1–7 in Q, heat flow at different conditions;
∞, undisturbed fluid condition;
d, based on gap;
x, local condition.
$'$ denotes differentation with respect to η;
— denotes dimensional quantities.

INTRODUCTION

HEAT TRANSFER to liquid metals flowing over various solids of different configurations is receiving considerable attention, with wider usages of liquid metals in space vehicles and nuclear reactors of different types. Free convection heat transfer becomes an area of major interest, in the case of failure of pumps or power in nuclear reactors as reported by Coombs *et al.* [1] and Hammit and Elayne [2].

Axial convection in a narrow sodium annulus has been investigated by Timo [3]. Schwab and De Witt [4] have reported the trend in the form of the variation of Nu with Gr and Pr, or Ra in the range of 2000–50,000 based on the finite difference solution of free convection heat transfer to non-metallic liquids in open top annuli, for Pr ranging from 0.7 to 7.0. Their results do not aim at a definite correlation, incorporating the geometry of the annuli. Nagendra *et al.* [5] have used for non-metallic fluids the concept of double boundary layer model of Emery and Chu [6], applicable for parallel plates, to derive correlations for vertical annuli in which the heat is transferred across the double boundary layer. They have reported the deviations of their experimental data on free convection heat transfer to water in long cylinders in annuli and also thin wires in annuli but they have not attempted any theoretical analysis of boundary layer equations for vertical annuli.

Nagendra and Tirunarayanan [7] have used the concept of a double boundary layer model similar to Emery and Chu [6] to solve the problem of free convection heat transfer between parallel flat plates under isothermal boundary conditions. Their analysis is not applicable to annuli, since the aspect of closed and open thermosyphons, mixing of boundary layers, effect of radius of curvature and the type of boundary conditions at the two bounding surfaces in annuli, offers challenging problems to the prospective investigators.

The ratio of heat transferred by convection to the heat transferred by pure conduction, as a function of Rayleigh number, has been suggested by Krussold [8, 9]. Free convection heat transfer data in annuli of Nagendra *et al.* [5] agree with Krussold's correlation. The comparison indicates that the correlations agree with the data for parallel vertical plates only, which is a special case of narrow annuli whose radial thickness is small. If the outer diameter of the annuli is increased, lower heat transfer rates are encountered.

Murakawa [11] has analysed the probem of free and forced convection heat transfer to non-metallic fluids in annuli and reported that *Nu* is proportional to the product of *Gr*, *Pr* and the gap to height ratio, as a special case for natural convection. Even the constant of proportionality has not been computed theoretically for non-metallic fluids. Since the thickness of the thermal boundary layer is quite appreciable for liquid metals whose *Pr* are small (the trend reported by Murakawa [11]) remains open for verification by rigorous free convection heat transfer data on liquid metals and theoretical analysis.

In the light of the above critical literature survey, it is very clear that there is hardly any information available on the theoretical analysis or the experimental data for the estimation of the rates of free convection heat transfer to liquid metals in isothermal vertical annuli with open tops.

It is the object of the present study to present a generalised theoretical approach for the estimation of free convection heat transfer rates to liquid metals based on a double boundary layer approach suggested by [6] and compare the present analysis with the existing theories and experimental data [4, 5, 8, 9] for non-metallic liquids. Further the present theoretical correlations will be compared with the present data on free convection heat transfer to water, and also the new data on free convection heat transfer to mercury in annuli of three radii ratios.

EXPERIMENTAL METHOD

In order to compare the theoretical results with the experimental data, a test rig is used in the present studies (Fig. 1). The geometries of the annuli and range of variables covered in the experiments are given in Table 1.

The test section was fabricated out of ES 304 Stainless Steel and consisted of two concentric tubes forming an annular gap. The surfaces of the tube were mirror polished by high speed buffing to even out the surface and minimize radiation losses. The outer tube was permanently fixed to a circular base and provisions were made to place the inner tube in a groove so that it was placed concentrically with the outer tube. An ebonite ring was used to bridge both the tubes at the bottom minimising conduction losses and serving as a guide to the inner cylinder. At the bottom of the tube a small circular reservoir was provided to contain the cold fluid while the open top of the annulus

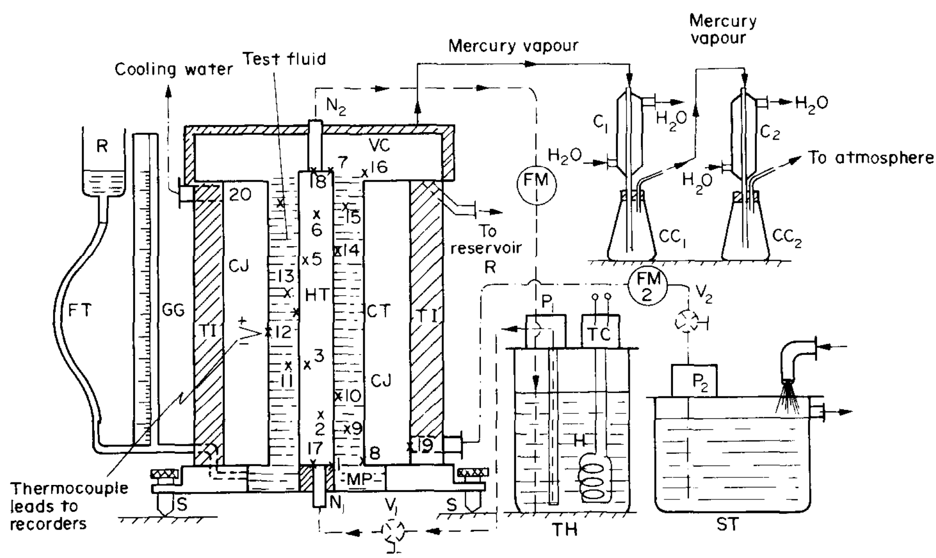

FIG. 1. Line diagram of test rig. HT: hot tube (source tube); CT: cold tube (sink tube); CJ: cooling jacket; MP: mercury pool; TI: thermal insulation; GG: gauge glass; FT: flexible tube; R: adjustable reservoir; VC: vapour chamber; N_1 N_2: nozzles; S: adjustable screw; V_1 V_2: control valves; C_1 C_2: mercury condensers; P_1 P_2: pumps; FM_1 FM_2: flow meters; H: heaters; TH: thermostat; TC: temperature controller; ST: cold water supply tank; C_1 C_2: condensate collectors: ------ heating circuit; —.—.— cooling circuit, ——— vapour line; X1 X2...... X20 thermocouple locations.

Table 1. Geometries of the annuli

Sl. no.	D_0 (cm)	D_1 (cm)	$C_R = D_1/D_0$	$d = (D_1 - D_0)/2$ (cm)	L (cm)	d/L	Test fluid	Range of Ra	Temperature rise (°C)
1	4.0	10.0	2.5	3.0	20.0	0.15	Water	940–4200	15–56.6
2	5.0	10.0	2.0	2.5	20.0	0.125	Water	540–2500	17–56.7
3	6.0	10.0	1.667	2.0	20.0	0.10	Water	230–1210	17.2–56.7
4	4.0	10.0	2.5	3.0	20.0	0.15	Mercury	150–520	10–33.8
5	5.0	10.0	2.0	2.5	20.0	0.125	Mercury	100–270	17.8–40.0
6	6.0	10.0	1.667	2.0	20.0	0.10	Mercury	30–125	15.1–44.8

was surrounded by a chamber which collected the overflow during thermosyphoning.

The inner tubes were fitted with inlet and outlet nozzles for passing hot water through the inner tube. The outer tube was surrounded by a jacket used for circulating a coolant. The entire apparatus was supported on three levelling screws of brass, which enabled the vertical position of the annular tubes to be maintained. The overflow chamber served as a vapour chamber and was closed with a top plate (which housed a top guide hole for the inner tube).

The inner tube was heated by the circulation of hot water supplied by a circulating thermostat which could be maintained at an accuracy of ± 0.1°C. Nozzles with a cylindrical neck were used to admit and withdraw hot water from the system. The nozzles were machined smoothly inside so that the flow rates were not altered. The flow rate of water was controlled by a wheel valve. Thus the walls of the annuli were maintained at a constant temperature by controlling the rates of flow of hot and cooling water. The tubes as well as the entire apparatus were completely lagged with asbestos rope and heat insulating cement, to minimise the heat losses.

The calibrated thermocouples were attached to the inner and outer tubes to measure the surface temperatures. The location of the thermocouples and the numbering scheme is illustrated in Fig. 2. The thermocouples were pulled out through holes drilled on the ebonite ring. The outputs of thermocouples were measured using a potentiometer of 0.0001 mV accuracy. A multichannel recorder was used with the aid of a selector switch, and precautions taken to place the thermocouples as close as possible to entry and exit points and recalibrate them *in situ*.

The apparatus was standardised by conducting several heat transfer experiments, filling the annulus with distilled water and taking measurements at various temperatures of the inner tubes. The equipment was taken to be standardized when the heat balance was about 95–98%. The same set-up was used to conduct experiments with mercury.

The experiments were conducted by filling the annular space with clean, commercially pure mercury and the level ascertained through the gauge glass. The contact thermometer in the circulating thermostat was adjusted to the desired value and the circulating thermostat switched on to a stabilized voltage a.c. supply. The flow rate of hot water was kept at certain value by adjusting the control valve. The outer tube was cooled simultaneously by forced flow around it, a desired rate of flow maintained and the recorders set in motion.

Procedures followed during the testing were the

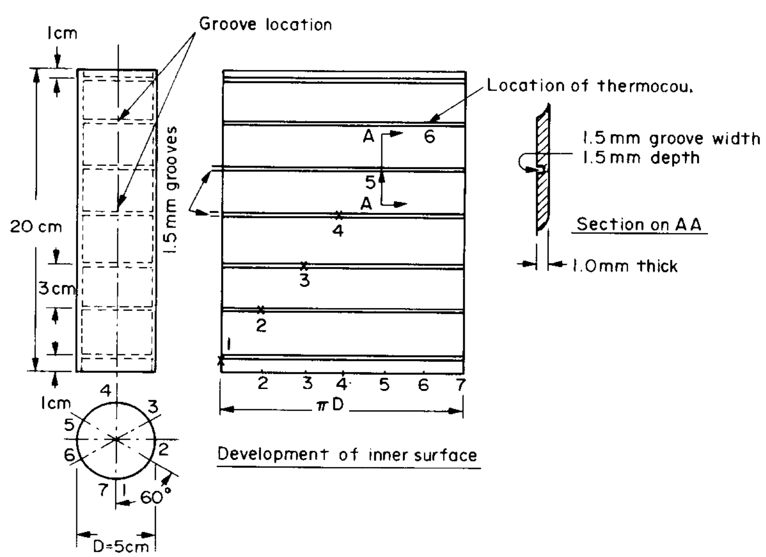

FIG. 2. Thermocouple location and numbering scheme.

same for all runs. As mentioned earlier the appropriate conditions for a run were established and the system allowed to reach steady state conditions. These were assumed when the system temperatures had stabilised at the point where indications on the source and sink tubes did not vary by more than 0.01 mV in a 30 min period. At this stage the inlet and outlet temperatures of the heating and cooling loops were also steady. In general 3–4 h of warm-up time was required to reach steady state during the runs with mercury.

After steady state had been reached, the temperatures of source and sink tubes, the rate of flow and the inlet and outlet temperatures of hot water, the rate of flow and the inlet and outlet temperatures of cold water were recorded.

It is best to use the bulk temperature in the computation of heat transfer coefficients, however the measurement of this by traversing thermocouples is beset with a number of problems. Firstly when slender probes are used, they bend in a vertical plane about the point of fixture owing to the high hydrostatic pressures of mercury. Thin glass probes easily break during traverse. Secondly when slightly thicker probes are used they disturb the natural convection flow because they act as obstacles. Thirdly when a probe is traversed in the field of mercury, fluctuations in the temperature arise during every movement and extra time will be necessary to reach steady state.

Lille and Nottage [10] have also encountered similar difficulties in their experiments on heat transfer to a liquid sodium–potassium alloy, contained between two parallel plates. They measured only the source and sink plate temperatures in order to compute bulk temperature as the average of these two. Such computation has been adopted by Dropkin and Somerscales [13] and Emery and Chu [14]. Hence, bulk temperature as an average of source and sink temperatures has been employed in the present studies.

The heat input and output of the test section were calculated using the rates of flow, specific heats and temperature differences. Using the above data, heat transfer coefficients were calculated as given below:

$$h = \frac{Q_6}{\pi D_0 L (T_1 - T_2)} \quad (1)$$

where the heat transfer coefficients are based on the length L. If they are based on the radial gap then L will be replaced by $d = (D_0 - D_1)/2$.

Q_6 is the total heat transferred by convection between two tubes $= Q_1 - Q_2 - Q_5$ where

(a) Q_1 is the heat lost by hot fluid in the test section which is given by

$$W_h \, C_p \, \Delta T_h.$$

(b) Q_2 represents the heat transferred by axial conduction through the wall of the source tube and is given by

$$-K_1 \, A_1 \, \Delta T_1 / L.$$

(c) Q_5 represents the heat transferred through radiation based on the surface area of the hot tube which is found to be negligibly small of the order of 0.005% of heat input and is estimated from

$$Q_5 = \pi D_0 L \sigma \varepsilon \bar{F}[(T_1/100)^4 - (T_2/100)^4] \cdot 10^8 \quad (2)$$

where σ is the Boltzman constant, ε is the emissivity and $\bar{F}$ is the configuration factor. The Nusselt number based on the diameter d is calculated using $Nu_d = hd/K_f$ and the Nusselt number based on the length L is given by hL/K_f where K_f represents the thermal conductivity of the fluid.

The heat flow by conduction alone is given by

$$Q_7 = 2\pi L K_f (T_1 - T_2)/\ln(D_1/D_0) \quad (3)$$

and the ratio of heat transferred by convection to the heat transferred by pure conduction is given by

$$K_e/K = Q_6/Q_7. \quad (4)$$

NUMERICAL SOLUTION

Statement of the problem

A vertical tube surrounded by another cylindrical tube concentric with the inner cylinder, containing liquid metal in the annular space bounded by the two cylindrical surfaces is the configuration used in the present studies. The inner cylinder was maintained at a constant wall temperature T_1, while the outer cylinder was maintained at a constant temperature T_2 which is lower than T_1. The top of the annulus was considered open and hence the pressure due to expansion forces of the liquid is negligible. The physical model and the system of coordinates were as shown in Fig. 3. The fluid rose along the hot surface and descended along the cold surface.

Formulation of the mathematical model

The formulation of the mathematical model of the problem was attempted with the following assumptions:

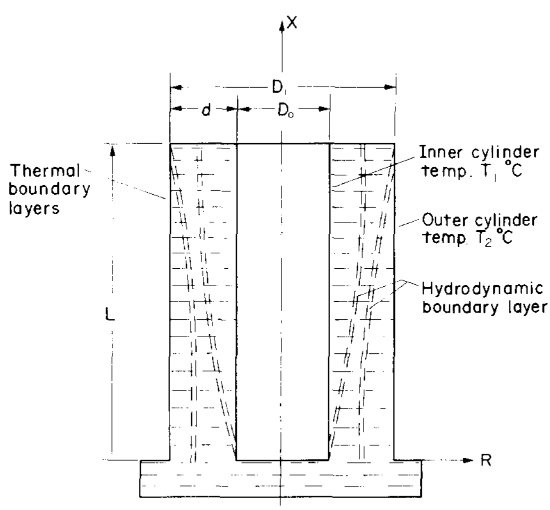

FIG. 3. Physical model and coordinates for vertical annuli.

(1) the fluid is incompressible;
(2) the generation of heat by friction, dissipation and effects of radiation are negligible;
(3) steady laminar axisymmetric conditions prevail;
(4) the surfaces are impermeable;
(5) the phenomenon of free convection in the enclosed annular space is a consequence of the coupled transfer of heat from the cylinder to fluid and from the fluid to the tube surface;
(6) the regime of the boundary layer flow exists in the laminar case and boundary layers do not mix within the annular space since the annular gap is sufficiently large compared with the boundary layer thickness;
(7) the radial velocities (R direction) are of order of magnitude smaller than the vertical velocities (X-direction);
(8) the gradients of velocity and temperature in the vertical direction are smaller than the gradients in the radial direction.

At this stage, if an order of the magnitude analysis of boundary layer equations is performed it is found that the conduction term in the axial direction, namely $\alpha\ (\partial^2 T/\partial X^2)$, is appreciable for liquid metals since the thermal boundary layer thickness for liquid metals is inversely proportional to $Pr^{1/3}$. In view of the non-mixing of the boundary layers for both surfaces, and the gradients of heat transfer in the X direction being smaller than in the R direction, the conduction term was neglected. In the present model however a new approach is proposed to account for the conduction. At some distance between the outer surface of inner cylinder and inner surface of outer cylinder, the temperature of the fluid is designated as T_∞. The process of free convection is now assumed to be due to the buoyancy from the temperature difference $(T - T_\infty)$. Further, towards the cold wall it is assumed that the process is governed by the same equations and satisfies outer velocity boundary conditions. At this juncture T_∞ will be considered as a variable to make T_2 constant. Hence the governing equations become:

$$\frac{\partial \bar{u}}{\partial X} + \frac{1}{R}\frac{\partial}{\partial R}(\bar{v}R) = 0, \tag{5}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial X} + \bar{v}\frac{\partial \bar{u}}{\partial R} = \frac{v}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \bar{u}}{\partial R}\right) + g\beta(T - T_\infty),$$

equation of motion (6)

$$\bar{u}\frac{\partial T}{\partial X} + \bar{v}\frac{\partial T}{\partial R} = \frac{\alpha}{R}\frac{\partial}{\partial R}\left(R\frac{\partial T}{\partial R}\right),$$

equation of energy (7)

with the boundary conditions $R = R_0$, $\bar{u} = \bar{v} = 0$ and $T = T_1$, at $R = R_1$, $\bar{u} = \bar{v} = 0$ and $T = T_2$ as given in Fig. 3.

Introducing the following dimensionless groups:

$$x = \frac{X}{d},\quad r = \frac{R}{d},\quad u = \frac{\bar{u}d}{v},\quad v = \frac{\bar{v}d}{v},$$

$$Pr = \frac{v}{\alpha},\quad Gr_d^+ = \frac{g\beta d^3(T_1 - T_\infty)}{v^2},$$

$$A = \frac{T_2 - T_\infty}{T_1 - T_\infty},\quad \theta = \frac{T - T_\infty}{T_1 - T_\infty},$$

$$d = (D_0 - D_1)/2 = (R_0 - R_1),$$

$$\eta = (Gr_d/32)\, r^2/x^{1/2},\quad \psi = 4xf(\eta),$$

$$t = \theta(\eta),$$

where η is the similarity variable and ψ is the stream function, equations (6) and (7) reduce to

$$f''' + f''(1 + 2f) - (f')^2 + \theta = 0 \tag{8}$$

$$\theta'' + \theta'(1 + 2f - Pr) = 0 \tag{9}$$

with the boundary conditions

$$\eta = \eta_{w_1} = \left(\frac{Gr_d^+}{32}\right)^{1/2} = \frac{D_0^2}{4d^2x^{1/2}};\quad f = f' = 0,\quad \theta = 1$$

$$\eta = \eta_{w_2} = \left(\frac{Gr_d^+}{32}\right)^{1/2} = \frac{D_1^2}{4d^2x^{1/2}};\quad f = f' = 0,\quad \theta = \mathrm{A}$$

$$\eta_{w_2}/\eta_{w_1} = D_0^2/D_1^2 = C^2. \tag{10}$$

The expressions for Gr and Nu are given by

$$Gr_d = \frac{g\beta d^3(T_1 - T_2)}{v^2} = Gr_d^+(1 - A), \tag{11}$$

$$Nu_d = -(D_1/L)(Gr_d^+/32)\int_0^{L/d}(1 - A)^{-3/2}\theta' \times (\eta_{w_1})x^{-1/2}\,dx. \tag{12}$$

RESULTS AND DISCUSSION

Equations (8) and (9) were solved numerically on IBM 360/44 computer using Runge–Kutta Gill method with Newton–Raphson modification for $Pr = 0.01$, 0.03, 0.733 and 5.0. The range of Pr for many liquid metals falls within 0.01 and 0.03. The outer boundary conditions are satisfied to an accuracy of 0.01% for various values of η_{w_1} ranging from 1 to 10 and radii ratios (C) $\sqrt{3}$, $\sqrt{4}$ and $\sqrt{5}$.

The variations of $\theta'\ (\eta_{w_1})$ with η_{w_1} and $(1 - A)$ with η_{w_1} for different Pr are represented in Figs. 4 and 5. In each case the variations can be represented by a pair of straight lines, whose slopes, indicated by numbers on the lines, change at a particular value of η_{w_1}. The critical value η_{w_1} refers to the points at which the region of conduction (represented by lines towards the left of this point) and the region of free convection (represented by the lines towards the right of this point) intersect. In each case the correlations are of the form:

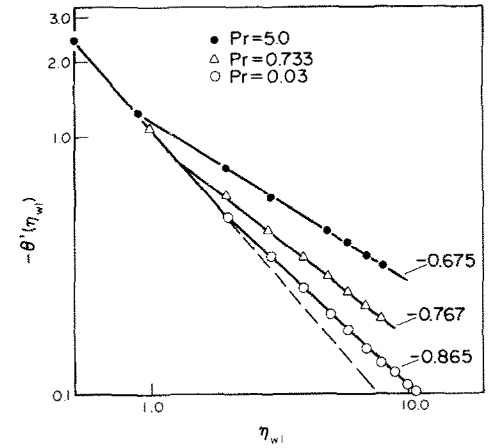

FIG. 4. Effect of Pr on $-\theta'(\eta_{w_1})$ at $C = 2.0$.

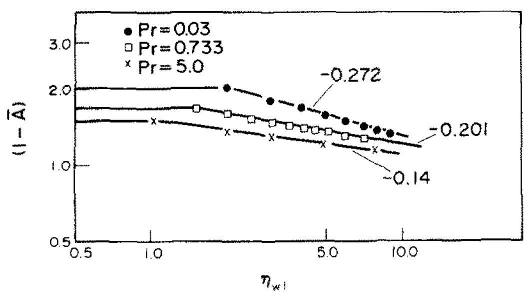

FIG. 5. Effect of Pr on $(1 - \bar{A})$ for $C = 2.0$.

$$\theta'(\eta_{w_1}) = P(\eta_{w_1})^p, \tag{13}$$

$$(1 - A) = S(\eta_{w_1})^s. \tag{14}$$

The exact relationships are obtained from the numerical computations as outlined above for each parameter and plotting the results (Figs. 4, 5).

The correlation of Nu by definition is then:

$$Nu_d = \left[\frac{PS^{(-3-p)/(2+s)}(2+s)}{4^{(-3s+2p)/(2+s)}(2s-p+1)}\right]\left(\frac{Gr_d}{32}\right)^{(1+p-s)/(2+s)}$$

$$(D_0/L)^{(2+4p-6s)/(2+s)}\,(d/L)^{(4s-3p-1)/(2+s)}. \tag{15}$$

On substitution for values of slopes and intercepts, it is observed that Nu is independent of Gr in the conduction region which is in thin layers, whereas in the convection region, Nu strongly depends upon Gr and gap to height ratio while it is independent of the (D_0/L) ratio. The constant term in the above equation is a strong function of Pr and is given by

$$F = 1.46(Pr)^{0.18}. \tag{16}$$

The final correlation can be written as

$$Nu_d = 1.46\,(Gr_d/32)^{0.25}\,(d/L)^{0.25}\,(Pr)^{0.18} \tag{17}$$

for $50 < Gr_d < 50{,}000$, $0.01 < Pr < 5.0$, $0.1 < d/L < 1.0$.

The powers of Gr_d and (d/L) are obtained by averaging the numerical solutions of differential equations (8) and (9).

Although Murakawa [11] has pointed out that Nu_d is proportional to:

$$(Gr_d)^{1/4}\,(d/L)^{1/4}\,(Pr)^{1/2} \tag{18}$$

for higher Pr, no attempt has been made to compute the proportionality constant. The power of Pr in the present analysis is lower than in the case of Murakawa [11] since his analysis is applicable to higher Pr.

The equipment has been used to collect data on heat transfer coefficients with water. The results of these experiments are reported elsewhere [15]. The experimental data are compared with the theoretical values and an error analysis for such data indicates a maximum deviation $\pm$ 12.5%. The comparison of the theory with the experimental data on water-filled annuli is indicated in Fig. 6. There are no published data either experimental or theoretical, in the literature, to check the validity of the present analysis for liquid metals. Hence the present experimental data on mercury-filled annuli and the theoretical analysis applicable for the lower ranges of Pr (liquid metals) are compared. The computations are given in [8–10]. The agreement between the theory and experiment is found to be good, as indicated in Fig. 7. The average deviation between the experimental results for mercury and theoretical results is 9% and maximum deviation is $\pm$ 13.7%. Hence the reliability of the correlation is established.

Schwab and De Witt [4] have tabulated Nu for Pr = 0.73, 1.0 and 7.0 for Ra in the range of 2000–50,000 but they have not given any definite correlation. The

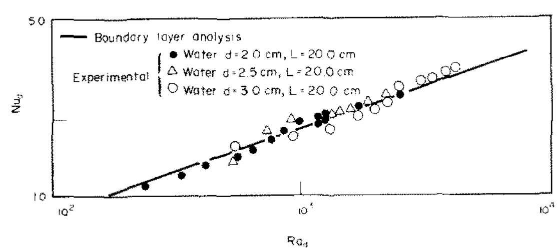

FIG. 6. Comparison of results for water-filled annulus.

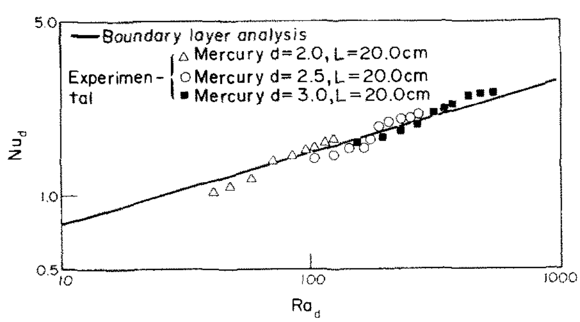

FIG. 7. Comparison of results for mercury-filled annulus.

present analytical results are found to be in good agreement with the finite difference solutions of [4] for the entire range. The comparison of the present experimental data with their theoretical analysis is also shown in Fig. 8 and found to be good.

Nagendra *et al.* [5] have reported correlations for the Nusselt number of closed top annuli formed by small tubes and wires. With long vertical annuli having large 'L/d' ratios, the radial heat transfer coefficients in closed top annuli do not vary appreciably when compared with those of open top annuli, as indicated by the expressions given by Fair Brother [12]. Hence the present analytical and experimental results are compared with those of [5] also. The agreement between the two results is found to be good at high Ra. The comparison between the present analytical results with the analytical as well as experimental results of [5] are shown in Fig. 9. The theoretical correlation for Nu_L in the present case is given by

$$Nu_L = 1.46\,(Gr_L/32)^{1/4}\,Pr^{0.18}. \tag{19}$$

Comparison of (K_e/K) *ratio*

The expression for K_e/K ratio is given by

$$\frac{K_e}{K} = \frac{\text{Heat transferred through convection}}{\text{Heat transferred through conduction}}$$

$$= \frac{Q_6}{Q_7} = \frac{h\pi D_0}{K_f \ln (D_1/D_0)} = \frac{(Nu)_L(D_0/L)}{\ln C}.$$

The values of K_e/K are found experimentally by measuring Q_6 and Q_7 as explained under 'Experimental method'.

The theoretical values of K_e/K are obtained from the above analysis by computing Nu_L for a given annulus using equation (19) and the value of Pr. While computing Nu_L, Gr will be based on the length. The theoretical and experimental values of K_e/K are plotted against Ra_d in Fig. 10, for non-metallic fluids and liquid metals.

It can be found from Fig. 10 that separate lines are obtained for non-metallic fluids and liquid metals. Krussold correlation [8, 9] (denoted by a broken line)

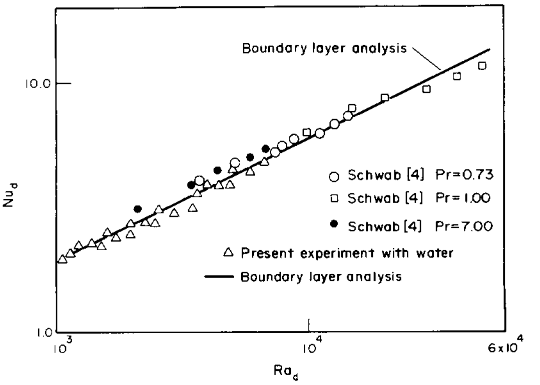

FIG. 8. Comparison of present theory with that of [7].

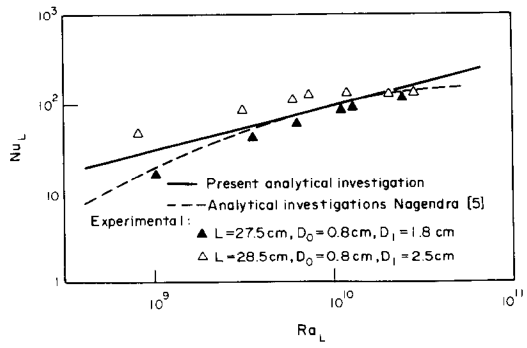

FIG. 9. Comparison of Nu_L with those of [5].

correlates the present data on water and the data of [5]. The solid line represents the values of K_e/K from theoretical analysis for non-metallic fluids like water.

The present experimental data on water and those of [5] agree very closely with the theory and also Krussold correlation. The correlation for non-metallic fluids can be written as

$$K_e/K = 0.168\,(Ra_d)^{0.287} \text{ for } Pr = 5.0,$$
$$10^2 < Ra_d < 10^5 \tag{20}$$

Since neither theoretical analysis nor experimental data are available for estimating K_e/K for liquid metals, the present theory for liquid metals is represented in Fig. 10 and present experimental data on mercury are compared with the theory. It can be found that the agreement between theory and experiment is excellent. Hence a general correlation for liquid metals can be written as

$$K_e/K = 0.281\,(Ra_d)^{0.284} \text{ for } Pr = 0.03,\ 10 < Ra_d < 10^3. \tag{21}$$

Correlations involving Rayleigh number

The present experimental data on water and mercury are computed as Nu based on gap width and plotted against Ra_d. d/L in Fig. 11. It is found that the present theoretical analysis can predict the experimental data very closely for the wider ranges of $Ra_d\ d/L$. The present experimental data confirm the theoretical analysis both for liquid metals and water. Hence the correlation can be written as

$$Nu_d = 0.761\left(Ra_d\frac{d}{L}\right)^{0.283},\ 10 < Ra_d\frac{d}{L} < 10^3, \tag{22}$$

$$Nu_d = 0.398\left(Ra_d\frac{d}{L}\right)^{0.284},\ 10 < Ra_d\frac{d}{L} < 10^5. \tag{23}$$

CONCLUSIONS

From these investigations and the analysis it can be concluded that:

(1) The boundary layer analysis, with the double

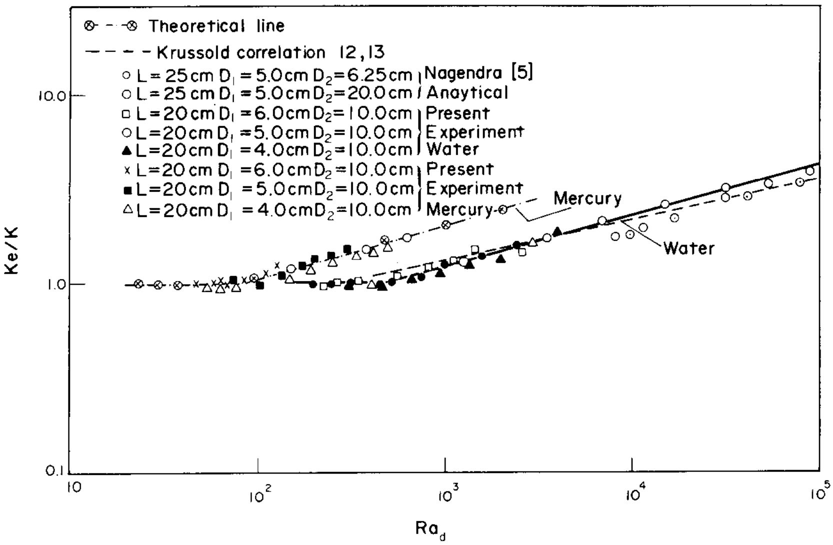

FIG. 10. Comparison of K_e/K with Krussold correlation [12, 13].

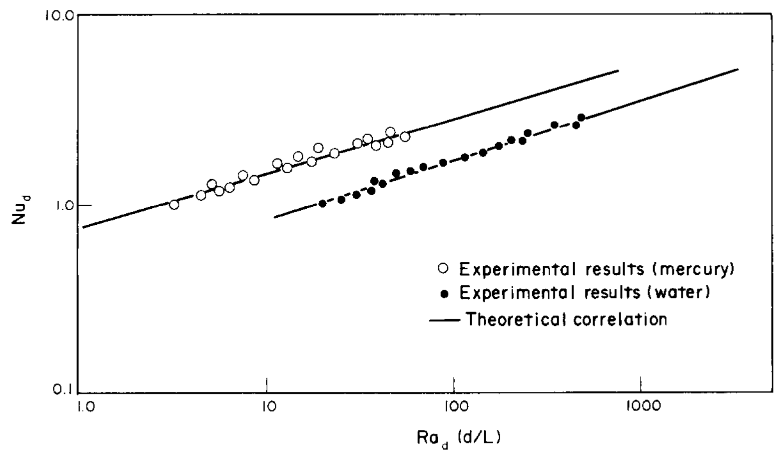

FIG. 11. Variation of Nu_d with $Ra_d(d/L)$.

boundary layer model considered in present studies is of comparable accuracy with the finite difference solutions.

(2) The value of η_{w_1} at which the plot of $-\theta'(\eta_{w_1})$ against η_{w_1} changes slope is capable of differentiating the conduction zone from convection zone.

(3) The theoretical correlations obtained are verified through experiments with mercury and water and are found to be in good agreement with present experimental data as well as those available in literature.

(4) The correlations obtained are capable of predicting heat transfer coefficients of both metallic and non-metallic fluids (in the Pr range 0.01–5.0) over a wide range of Ra within a deviation of 9% and maximum deviation of $\pm$ 13.7%.

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CONVECTION NATURELLE THERMIQUE POUR LE MERCURE DANS UN ESPACE ANNULAIRE VERTICAL

Résumé—Des données sur la convection naturelle thermique pour l'eau et le mercure sont obtenus à partir d'un montage d'essai correspondant à un espace annulaire avec trois rapports de rayons, les parois étant maintenues à des températures uniformes. Une analyse théorique des équations de couche limite a été traitée en utilisant une transformation de similarité. Des formules dérivées de cette analyse théorique sont comparées avec les données expérimentales et les résultats théoriques déjà connus pour des fluides non métalliques et aussi avec les données présentes sur l'eau et le mercure. Des formules généralisées sont posées pour exprimer le rapport de la chaleur transférée par transport à la chaleur transférée par conduction et on donne le nombre de Nusselt en fonction des nombres de Grashof, de Rayleigh et de Prandtl. Les formules généralisées s'accordent avec les données pour les fluides non métalliques et les métaux liquides avec une déviation moyenne de neuf pour cent et une déviation maximale de ± 13,7 pour cent.

WÄRMEÜBERTRAGUNG DURCH FREIE KONVEKTION BEI QUECKSILBER IN VERTIKALEN RINGSPALTEN

Zusammenfassung—Es wurden Daten zur Wärmeübertragung bei freier Konvektion von Wasser und Quecksilber in vertikalen Ringspalten ermittelt, wobei ein Versuchsaufbau mit drei Radienverhältnissen und gleichförmiger Wandtemperatur verwendet wurde. Mit Hilfe lokaler Ähnlichkeitstransformation und zweifacher Grenzschichtnäherung wurde eine theoretische Betrachtung der Grenzschichtgleichungen durchgeführt. Die aus der vorliegenden theoretischen Behandlung abgeleiteten Beziehungen wurden sowohl mit den in der Literatur vorhandenen Theorien und experimentellen Daten für nichtmetallische Fluide als auch mit den eigenen Meßwerten für Wasser und Quecksilber verglichen. Aufbauend auf der theoretischen Untersuchung und den ermittelten Daten für Quecksilber und Wasser wurden für das Verhältnis der durch Konvektion zu der durch reine Leitung übertragenen Wärme und für die Nusselt-Zahl allgemeine Beziehungen in Abhängigkeit von Grashof-, Rayleigh- und Prandtl-Zahl aufgestellt. Die angegebenen allgemeinen Beziehungen stimmen mit den zitierten Daten und den eigenen Werten für nichtmetallische Fluide und flüssige Metalle bei einer mittleren Abweichung von 9% und einer maximalen Abweichung von ± 13,7% überein.

СВОБОДНОКОНВЕКТИВНЫЙ ТЕПЛОПЕРЕНОС К РТУТИ В ВЕРТИКАЛЬНЫХ КОЛЬЦЕВЫХ КАНАЛАХ

Аннотация — Представлены данные по свободноконвективному теплопереносу к воде и ртути, полученные на вертикальных кольцевых каналах с тремя различными отношениями радиусов при постоянной температуре стенок. С помощью локального автомодельного преобразования и приближения двухслойного пограничного слоя проведен теоретический анализ. Теоретически полученные соотношения сопоставлены с опубликованными аналитическими и экспериментальными данными для неметаллических жидкостей, а также с данными, полученными в настоящей работе для воды и ртути. На основе теоретического анализа и экспериментальных данных для воды и ртути предложены обобщенные соотношения для расчета отношения количества тепла, переданного конвекцией, к количеству тепла, переданного только теплопроводностью, и определения числа Нуссельта, зависящего от чисел Грасгофа, Релея и Прандтля. Полученные обобщенные соотношения описывают опубликованные данные и данные настоящей работы для неметаллических жидкостей и жидких металлов со средней погрешностью в 9% и максимальной в ±13,7%.